More regressions stuff!

Econ 140, Section 4

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- 1. Omitted Variable Bias
- 2. Bad Controls
- 3. Quadratic Terms
- 4. Interaction terms

Any questions?

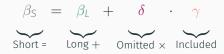
... The midterm is getting closer: Now is the time for clarifying questions!

Omitted Variable Bias

We can summarize everything of OVB in three equations. Let *Y_i* be the outcome variable, *X_i* our regressor of interest, and *Z_i* the "omitted" variable.

[Long regression] $Y_i = c_1 + \beta_L X_i + \delta Z_i + e_i$ [Short regression] $Y_i = c_2 + \beta_S X_i + u_i$ [Auxiliary regression] $Z_i = c_3 + \gamma X_i + v_i$

Then, the Omitted variable bias formula states that:



We call $\delta\gamma$ the **omitted variable bias**. We can appraise the direction of the bias by multiplying our guesses for the signs of δ and γ . If either $\delta = 0$ or $\gamma = 0$, then OVB is zero!

Regression: Basics, Interpretation, Quadratic Terms

See in Datahub: OVB example, OVB visualized

Bad Controls

Control variables are additional variables (or covariates) included in a regression. We do this for various reasons (in decreasing order of importance):

- To remove selection bias / omitted variable bias
- To increase precision of our estimates
- To know about the (conditional/partial) correlation of other variables
- To better predict the outcome

Let's see graphically how control variables work! Datahub

- Some controls are called "bad controls". These are:
 - 1. Variables that are themselves outcomes of a treatment: What happens if you control for the change in English test scores in the regression below?
 - 2. Variables that moderate the treatment effect, e.g. controlling for occupation choice in gender wage gap regression ...
- Rule of Thumb: Good controls are either pre-determined or immutable characteristics.
- Another way to think about it: Controls help us make "apples to apples" comparisons. Which apples matter?

Mathematically, good and bad controls are the same thing. We need to use our 🖘 to distinguish them!

Quadratic Terms

Making OLS more interesting

• We can extend the simple OLS framework

 $Y_i = \beta_0 + \beta_1 X_i + e_i$

to something richer:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + e_i$$

 All questions of the type "how is Y_i expected to change if we change X_i", keeping all other variables in the regression fixed can be solved with partial derivatives – in this case:

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$$\frac{\partial Y_i}{\partial X_i} = \beta_1 + 2 \cdot \beta_2 \cdot X_i$$

Let's see how to use quadratic terms on **Datahub**

Interaction terms

Interaction terms: Making OLS more interesting

Let us consider the model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + e_i$$

where Y_i is a country's GDP per capita, X_{1i} the value of its natural resources, and X_{2i} a measure of how democratic it is.

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Keeping democracy fixed, increasing the value of a country's natural resources by one unit is associated with β_1 higher GDP per capita.

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2. How do we interpret β_2 ?

Keeping natural resources fixed, increasing a country's democracy score by one unit is associated with β_2 higher GDP per capita.

Now, let us extend the model to:

$$Y_{i} = \beta_{0} + \beta_{1}X_{1i} + \beta_{2}X_{2i} + \beta_{3}X_{1i}X_{2i} + e_{i}$$

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- 3. How do we interpret β_2 ? The effect of an additional unit of X_{2i} , if X_{1i} is equal to 0.
- 4. How do we interpret $\beta_1 + \beta_3$? The effect of an additional unit of X_{1i} , if X_{2i} is equal to 1.

Rule of thumb: Always use partial derivatives to make sure that you are right!

Interactions with only dummy variables

- Take any two binary variables, for example: Studies at UC Berkeley or not (UCB_i), and is from California or not (Cali_i).
- The regression with interactions looks like this:

$$Y_{i} = \underbrace{\beta_{0}}_{17.77} + \underbrace{\beta_{1}}_{2.28} UCB_{i} + \underbrace{\beta_{2}}_{0.95} Cali_{i} + \underbrace{\beta_{3}}_{-0.97} UCB_{i} \times Cali_{i}$$

• We can write this in a table, and get all group averages

	Cali = 1	Cali=0
UCB = 1	$\beta_0 + \beta_1 + \beta_2 + \beta_3$ =17.77+2.28+0.95-0.97	$\beta_0 + \beta_1$ = 17.77 + 2.28
UCB = 0	$\beta_0 + \beta_2$ = 17.77 + 0.95	β ₀ =17.77

• Take differences between cells to get different effects!